Evaluation the Effect of Residual Stress on Fracture of Polyethylene Pipe under Pressure Loading

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***Abstract* - In this paper, the Weibull uni-axial and multi-axial distribution function for polyethylene pips under pressure loading were developed and analyzed taking account of residual stress. Tensile test was achieved to determine mechanical properties and the Weibull parameters. Experimental method using the hole-drilling strain-gage method was used to measure the residual stresses in PE pipe and compare with that obtained from numerical finite element method (FEM). The obtained results show that there is a convergence between uni-axial and multi-axial distribution function, but multi-axial distribution function give large values compared to uni-axial distribution function. It was observed that the residual stresses have influence on failure assessment diagram and causes translation from elastic-plastic failure to brittle failure.**

Key words: Probability of failure, Weibull parameters, FAD diagram, Residual stress, Cracks

I. Introduction

Polyethylene (PE) is the preferred material used for water distribution pipeline because it's has low price, light weight, ease of joining and installed, long-term durability and flexibility. The internal and external defects, residual stresses, high temperature and high pipeline pressure are major failure reasons of water pipelines [1].

All PE pipes contain residual stresses that creating during their manufacture or processing. The state of residual stress created varies depending on the types of processing conditions like melt temperature, shearing and cooling rates. Also the residual stresses induced by pressure loading during service. Residual stresses determined by three methods: analytical, computational with finite element method FEM and experimental methods [2].

The failure of PE pipe are obtains using various methods especially using Weibull analysis and failure assessment diagram (FAD) [3]. Weibull distribution is flexible and use two or three parameters: scale, shape and location parameters. There are a number of methods for estimating the values of these parameters. Some of these methods are graphical include plotting and others are analytical includes maximum likelihood, least square and moment's method [4].

The failure assessment diagram (FAD) is a graphical plot of the failure based on elastic-plastic concepts. The FAD uses of two parameters K_r and L_r and can be applied to determine the probability of failure, i.e., if the extension crack in pipe is safe or not [5].

Various studies in PE pipe are achieved. Broutman et *al.* [6] compares three methods: slitting, Turing (boring) and hole drilling methods for measuring residual stress in plastic pipes. They show that the three methods can be used to measure residual stresses and give good agreement. Kaelif et *al.* [7] developed reliability model for analysis of HDPE gas pipeline taking account of residual stresses. They show that the information produced by the reliability assessment is a better tool for pipe inspection and optimization. Joon et *al.* [8] estimation the failure probability of the pressure tubes using probabilistic fracture mechanics. They proposed FAD of pressure tube material and applied in the probabilistic analysis. They show that the dimensional changes have high sensitivity for failure probability analysis. Frank et *al.* [9] measure residual stresses in PE pipe after 30 year of uses. Their results show that the magnitude of residual stresses is significant in comparison with hoop stresses induced by inner pressure. Alexander et *al.* [10] formulated the probability of the crack initiation using the statistical fracture mechanic and chemical degradation induced stresses. They determined the critical level of degradation by using point-wise Weibull distribution. Hutar et *al.* [11] study by using both experimental and finite element method the effect of residual stresses on crack geometry and behavior of polymer pipes. It was found that the crack shape was influenced by the presence of residual stresses. O'Connor et *al.* [12] study and review the pressurized PE pipe failures. They show that the failure modes of PE pipe under internal pressure is three modes, ductile, brittle and brittle–ductile failures. Tee et *al.* [13] used method of subset simulation for time dependent reliability prediction of the underground pipelines. They show that this method can provide better resolution for low of probability of failure. Hanl *et al.* [14] presents an analysis of failure assessment diagram for PE4710 grade high density PE pipes used in the nuclear industry. It was verified that failure mode of PE pipes can be predicted through the determination of critical crack size in FAD framework.

In this paper, the Weibull uni-axial and multi-axial distribution functions are developed and used to analysis of PE pipe to evaluate the reliability of fracture. Tensile test is achieved to evaluate PE pipe material properties and Weibull parameters. the hole-drilling strain-gage method was used to measure the residual stresses in PE pipe and compare with that obtained from numerical finite element method (FEM) using the commercial software Ansys V6.7.

II. Numerical Methods

(a) Finite element analysis (FEM)

Elastic-plastic finite element method (FEM) was used to estimate the residual stresses in the case of PE pipe under pressure loading. FEM includes three steps [15]: the region of domain is discretize to fixed form elements, the formation of these elements governing equations and then solve equations to get the result after applying the boundary conditions. In this research used special program Ansys software V6.7. Due to the symmetry, only one half of PE specimen may be modeled in 3-D plane strain. The finite element mesh of specimen using 3-nodes element were implemented in the analysis and shown in Fig.1.

The total numbers of element used for analysis are 1000 elements with 1133 nodes. The boundary conditions are all internal nodes of internal boundary elements of pipe are subjected to a pressure loading of 5 MPa and all other nodes free stresses in y and z direction. According to pressure technology association code of practice the plastic zone extends to the geometrical mean radius $r_p = \sqrt{r_i r_o}$ [15] and the plastic pressure is calculated using the following equation [14]:

$$P_{Ps} = \frac{\sigma_y}{2r_o^2}\left(r_o^2 - r_p^2\right) \quad (1)$$

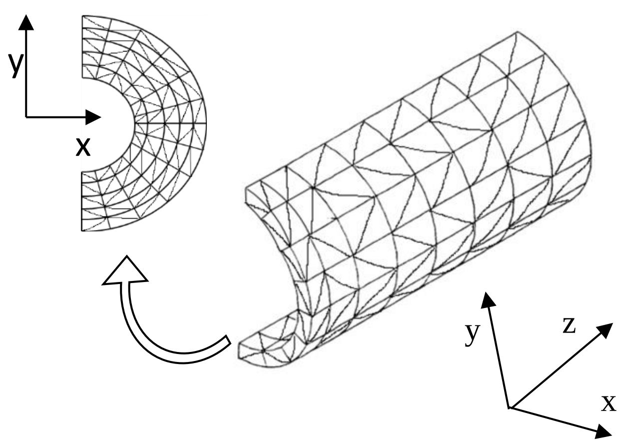

Fig.1 3D-finite element mesh using 3-nodes element

(b) The structural integrity assessment procedure (SINTAP)

The SINTAP for European industry procedure offers a good approach to draw failure assessment diagram FAD. The procedure is explained as followers [17]:

The basic equation of SINTAP-FAD route:

$$K_r = f(L_r) \quad (2)$$

Where,

$$K_r = \frac{K_T}{K_{mat.}} \quad (3)$$

The function $f(L_r)$ is given as:

$$f(L_r) = [1 + 0.5L_r^2]^{-0.5} \ \text{for} \ \ 0 \le L_r < 1 \quad (4)$$

And

$$f(L_{r=1}) = \left[\lambda + \frac{1}{2\lambda}\right]^{-0.5} \text{for} \ \ L_r = 1 \quad (5)$$

Where

$$\lambda = 1 + \frac{E\Delta\varepsilon}{\sigma_y} \quad (6)$$

And

$$f(L_r) = f(L_{r=1}) \times L_r^{(n-1)/2n} \quad \text{for} \ \ 1 \le L_r < L_{r,max.} \quad (7)$$

The Luders strain is estimated from an empirical equation as follows:

$$\Delta\varepsilon = 0.0375\left[1 - \frac{\sigma_y}{1000}\right] \quad (8)$$

The strain-hardening exponent is obtained using the empirical relation:

$$n = 0.3\left(1 - \frac{\sigma_y}{\sigma_u}\right) \quad (9)$$

The plastic collapse load is defined as:

$$L_{r,max} = \frac{1}{2}\left[\frac{\sigma_y + \sigma_u}{\sigma_y}\right] \quad (10)$$

Fig. 2 shows the general FAD using the SINTAP approaches and division of regions [18]. Based on Feddersen [19] divisions of regions, the limits of these three regions are defined as follows:

Zone I : $0 < L_r < 0.62$ (LEFM)
Zone II : $0.62 < Lr < 0.95$ (EPFM)
Zone III : $0.95 < L_r < L_{r,max}$ (limit load)

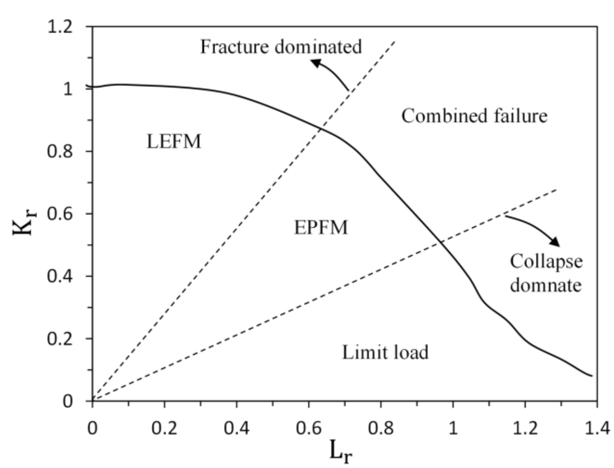

Fig. 2 The general FAD zone [16]

(c) Stress intensity factor and residual stress

The strength of PE pipe is reduces when cracks are present through thickness because the stresses and strains are highly magnified at the crack tip. The use of parameter to describes the local stresses and strains at crack tip are important to evaluation the structural integrity. This parameter is called stress intensity factor K_I. The values of K_I are influence by the presence of residual stresses.

For the cracked PE pipe loaded by internal pressure, the stress intensity factor in opening mode (mode I) was found used a semi-analytical expression developed by Hutar et al. [2] as follows:

$$K_{Iint} = \frac{P_{int} D_i}{t}\sqrt{\pi a}\, Y\left(\frac{a}{t}\right) \quad (11)$$

Where,

$$Y\left(\frac{a}{t}\right) = 0.3417 + 0.0588\left(\frac{a}{t}\right) - 0.0319\left(\frac{a}{t}\right)^2 + 0.1409\left(\frac{a}{t}\right)^3 \quad (12)$$

Eq. 11 does not consider into account the residual stress in the pipe wall. This, for determining the stress intensity factor considering the effect of residual stresses into account, the Eq.11 is modified as follows [2]:

$$K_{I\,(int+res)} = \frac{(P_{int}+P_{res}) D_i}{t}\sqrt{\pi a}\, Y\left(\frac{a}{t}\right) \quad (13)$$

Where,

$$Y\left(\frac{a}{t}\right) = 0.3417 + 0.0588\left(\frac{a}{t}\right) - 0.0319\left(\frac{a}{t}\right)^2 + 0.1409\left(\frac{a}{t}\right)^3 \quad (14)$$

The value of internal pressure that corresponds to the measured residual stress at the inner wall of PE pipe (P_{res}) obtains as flowers:

$$P_{res} = \frac{4}{3}\,\frac{\sigma_{res}\, t}{D_o - 2t} \quad (15)$$

(d) The Expected rupture strength of PE pipe under pressure loading

If consider the PE pipe is elastic, the radial and tangential stresses in pipe under internal pressure loading P are given as [20]

$$\sigma_r = \frac{P\, r_i^2}{r_o^2 - r_i^2}\left[1 - \frac{r_o^2}{r^2}\right] \quad (16)$$

$$\sigma_t = \frac{P\, r_i^2}{r_o^2 - r_i^2}\left[1 + \frac{r_o^2}{r^2}\right] \quad (17)$$

Under the boundary condition at $r = r_i$ then $\sigma_{max.} = \sigma_t$, this from Eq.17 obtains:

$$\sigma_{max.} = P * \frac{r_i^2 + r_o^2}{r_o^2 - r_i^2} \quad (18)$$

Two cases are considered to determine probability of failure of PE pipe:

Case 1

For polyethylene pipe has internal cracks and under pressure loading only for the state of uniaxial distribution function, defining a function $f1(r)$ as [20]:

$$f1(r) = \frac{\sigma_t}{\sigma_{max.}} = \frac{r^2 + r_o^2}{r_i^2 + r_o^2} * \frac{r_i^2}{r^2} \quad (19)$$

The probability of failure is given as:

$$R(\sigma_{max.}) = 1 - e^{-B_o\left(\frac{\sigma_{max.} - \sigma_u}{\sigma_o}\right)^m} \quad (20)$$

Where,

$$B_o = 2\pi L \int_r^{r_o} f(r)^m\, r dr \quad (21)$$

When taking the effect of residual stresses, the symbols $\sigma_{max.}$ become the maximum applied stresses plus residual stresses measured. In most cases $\sigma_u = 0$.

Case 2

For polyethylene pipe has internal cracks and under pressure loading, the probability of failure for the state of multi-axial distribution function, derived as follower:

Defining a function $f_2(r)$ as:

$$f2(r) = \frac{\sigma_r}{\sigma_{max.}} = \frac{r^2 - r_o^2}{r_i^2 + r_o^2} * \frac{r_i^2}{r^2} \quad (22)$$

The probability of failure is given by the following equation [21]:

$$R(\sigma_{max.}) = 1 - e^{-B_1\left(\frac{\sigma_{max.} - \sigma_u}{\sigma_o}\right)^m} \quad (23)$$

Where,

$$B_1 = \phi \int_{r_i}^{r_o} \int_0^{\frac{\pi}{2}} f1(r)\cos^2\phi + f2(r)\sin^2\phi]^m\, r d\phi dr \quad (24)$$

Where,

$$\phi = \frac{4\pi L}{\beta\left(m + \frac{1}{2}, \frac{1}{2}\right)} \quad (25)$$

The function $f1(r)$ is given by Eq.19, whiles the two parameters K and β are given by the following equations [22]:

$$K = (\beta\, \sigma_o^m)^{-1} \quad (26)$$

$$\beta\,(\chi, \lambda) = \int_0^1 X^{\chi-1}(1-X)^{\lambda-1} dX \quad (27)$$

And

$$\chi = m + 0.5\,, \quad \lambda = 0.5$$

(e) Method of estimation the Weibull parameters m and σ_o

One of the best methods to determine the Webull parameters m and σ_o used data obtained from tensile test is the least square method. Consider $\sigma_u = 0$, $\sigma_{max.} = \sigma$ is applied stress and B_o is the surface area of specimen. Then the probability of failure, i.e. Eq.20 can be written as [23]:

$$R(\sigma) = 1 - e^{-B_o\left(\frac{\sigma}{\sigma_o}\right)^m} \quad (28)$$

Rearrangement and take ln for two sides

$$1 - R(\sigma) = e^{-B_o\left(\frac{\sigma}{\sigma_o}\right)^m} \quad (29)$$

$$\ln(1 - R(\sigma)) = -B_o\left(\frac{\sigma}{\sigma_o}\right)^m \quad (30)$$

$$\ln(-\ln(1 - R(\sigma))) = m\ln B_o + m\ln\sigma - m\ln\sigma_o \quad (31)$$

$$\ln(-\ln(1 - R(\sigma))) = m\ln\sigma - m\ln B_o + m\ln\sigma_o \quad (32)$$

Eq.32 can be written as following:

$$Y = bX + c \quad (33)$$

Where,

$$Y = \ln(-\ln(1 - R(\sigma))) \quad (34)$$

$$X = \ln\sigma \quad (35)$$

$$b = m \quad (36)$$

$$c = m\,(-\ln B_o + \ln\sigma_o) \quad (37)$$

Then can be used the data obtained from tensile test to obtain the values of constants c and b by using least square method, i.e. by using the linear regration formula [24]:

$$b = \frac{n\sum_{i=1}^{n} X_i Y_i - \sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i}{n\sum_{i=1}^{n} X_i^2 - \left(\sum_{i=1}^{n} X_i\right)^2} \quad (38)$$

$$c = \frac{\sum_{i=1}^{n} X_i^2 \sum_{i=1}^{n} Y_i - \sum_{i=1}^{n} X_i \sum_{i=1}^{n} X_i Y_i}{n\sum_{i=1}^{n} X_i^2 - \left(\sum_{i=1}^{n} X_i\right)^2} \quad (39)$$

By consider the tensile stress obtained from tensile test as Y and strain as values of X and n represent number of data points used for calculation. The values of constants c and b are obtained and then used Eqs.36 and 37 to obtain Webull parameters m and σ_o.

III. Experimental Approaches

(a) Hole-drilling method to determine residual stresses

In this method, a small hole was drilling in the PE pipe wall to a small depth Δh (typically range from 1 to 2 mm). Then the surface strain change was measured using strain gages rosette (±0.01 accuracy). The values of residual stresses in two dimensions obtained from the following equations [25]:

$$\sigma_{rr} = \frac{\varepsilon_r}{A + B*COS(2\alpha)} \quad (40)$$

$$\sigma_{\theta\theta} = \frac{\varepsilon_r}{-A + F*COS(2\alpha)} \quad (41)$$

Where,

$$A = -\frac{1+\nu}{2E}\left(\frac{1}{r_s}\right) \quad (42)$$

$$B = -\frac{1+\nu}{2E}\left[\left(\frac{4}{1+\nu}\right)\frac{1}{(r_s)^2} - \frac{3}{(r_s)^4}\right] \quad (43)$$

$$F = -\frac{1+\nu}{2E}\left[-\left(\frac{4}{1+\nu}\right)\frac{1}{(r_s)^2} + \frac{3}{(r_s)^4}\right] \quad (44)$$

And

$$r_s = \frac{r_o}{r_h} \quad (45)$$

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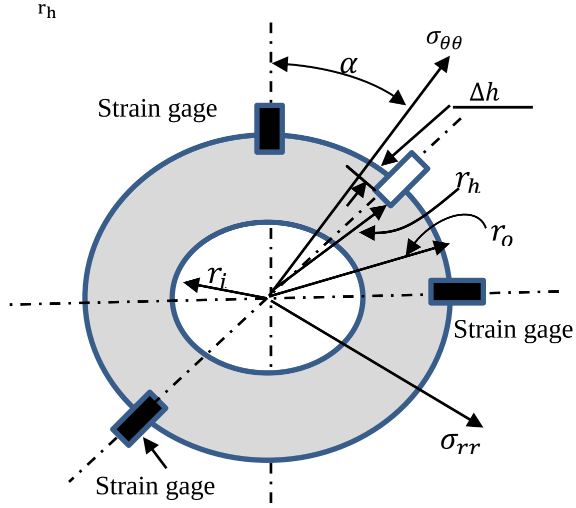

Fig. 4 Arrangement of the strain gage rosette for measuring the residual strain.

(b) PE Pipe specimen and tensile Test

The polyethylene pipe specimen is shown in figure 5 manufactured by Shandong Yanggu Hengtai Industrial Co., Ltd, China

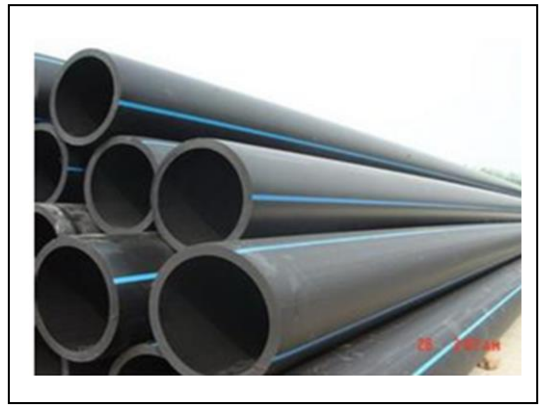

Fig. 5 Polyethylene pipe specimen.

The maximum pressure PE pipe withstands is 10 MPa. The pipes dimensions are: D_i = 27.43 cm, D_o= 30.48 cm, t = 15.25 mm and L = 6 m. The plain strain fracture toughness and Poisons ratio are: K_{ic}= 6 MPa. $\sqrt{m}$ and $\nu = 0.33$ respectively [26].

The specimens for tensile tests were machined from the polyethylene pipe materials according to the ASTM D638-03 standard method of test for tensile properties of plastics [27]. Figures 6 and 7 illustrated the specimen dimensions (mm) and shape of tensile tests. The uniaxial tension test of the polyethylene specimen was conducted on tensile testing machine (Hounsfield hand operated tensile machine SM1002, Hounsfield Company, UK).

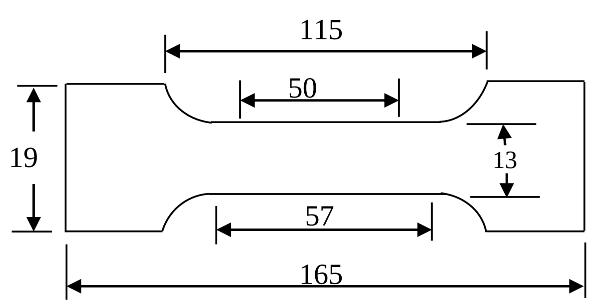

Fig. 6 Geometry and dimensions (mm) of the tensile specimen have Thickness 10 mm.

Fig. 7 Tensile test specimens.

IV. Results and Discussion

To determine the PE pipe material properties and Weibull parameters m and σ_o, tensile testing of the Polyethylene pipe was conducted. Figure 8 shows of force –elongation curve for PE pipe materials at temperatures 25ºC obtained from tensile test. As indicated, small extension with large force during the beginning of test, but after extension reaches 15 mm the force begins to decrease and large extension occurs until failure. The deformation occurs under applied tensile load was not uniform along the gauge length up to the fracture point. The maximum force occurs at of extinctions 8 mm and has a value of 12.8 kN. The stresses and strains are obtained from the figure 8. The applied stress is calculated from division of force to original sectional area and strain from division extensions to original length. The obtained data during the testing were used to establish ultimate strain, elastic moduli, yield stress and yield strain values for PE pipe materials. Then, the stresses and stain obtained used with least square method i.e. Eqs.29-32 and solved the equation to obtain Weibull parameters m and σ_o. The elastic constant and the Weibull parameter are summarized in table 1. Fig.9 show comparison of the tangential residual stresses distribution across the pipe wall thickness calculated using FEM and that obtained using experimental hole-drilling method. In FEM, the residual stress is the difference between elastic and plastic stress. As illustrate large differences between FEM and experimental method. This attributed to that the FEM based on numerical conditions and due to some approximation in the experimental method.

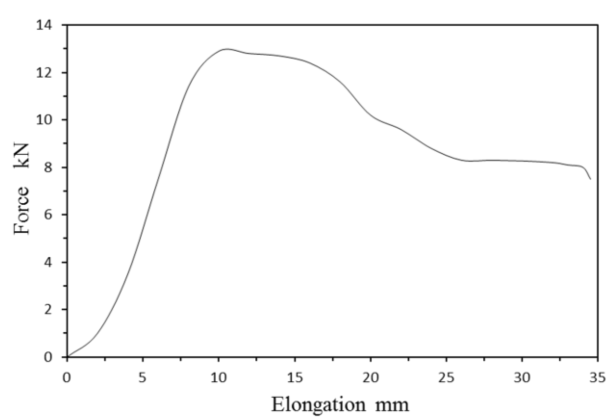

Fig. 8 Force –extension curve for PE pipe material.

Table 1 Weibull Parameters and tensile properties of PE.

T ℃	m	σ_o MPa	E GPa.	S_u MPa	S_y MPa
25	0.215	4.7	1.09	98.3	26

As seen in figure 9, the behaviors of residual stresses curves are difference between FEM and experimental methods. In a FEM is approximately linear while that for experimental method the residual stresses exhibited a parabolic shape curves. The values of tangential residual stresses from the FEM and experimental methods are 25.414 and 24.21 MPa at inside diameter, while the values are 17.56 and 16.43 MPa at outside diameter respectively. Generally, the absolute values of residual stresses in circumferential direction are much larger than those that in radial direction. The largest tensile of tangential residual stress are found on the inner surface and minimum value occurs at outside surface. This action is believed to be attributed to the cooling method used during the pipe manufacturing process in addition to the effect of the internal pressure loading that take through accounts.

In the hole-drilling method, the values of radial residual stresses is small and can not be measured, so the values of tangential residual stresses will be measured only and compared with that obtained from FEM using Ansys softwere. Table 2 shows the values of radial residual stresses obtained from FEM analysis. From table 2, the value of radial residual stresses obtained from the FEM at inside and outside diameters are -2.393 MPa and 0.135 MPa respectively. The largest tensile of radial residual stress are found on the outer surface, while inner surface have compressive stresses

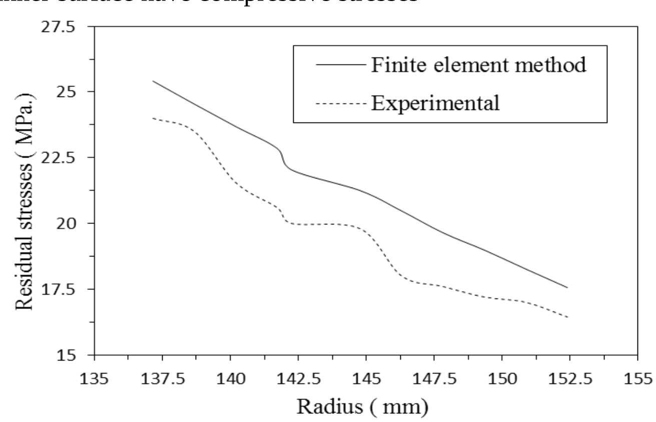

Fig. 9 Tangential residual stress distributions.

Table 2 Radial residual stresses calculation using FEM.

Radius mm	Radial residual stresses MPa.
137.15	-2.393
138.675	-2.093
140.2	-1.807
141.725	-1.538
142.25	-1.283
144.775	-1.041
146.3	-0.813
147.825	-0.597
149.35	-0.393
150.875	-0.201
152.4	0.135

To determine reliability of PE pipe, effect of residual stresses in probability of failure and if a crack may cause a PE pipe failure, the failure assessment diagram (FAD) is drawn in figure 10. The failure assessment curve is drawn from using the values of K_r calculated using Eq.2 and for a range values of L_r from 0 to $L_{r,max.}$ using Eq.3-9. The maximum values of Lr (plastic collapse load) obtained using data of table 1 and use Eq.10 is equal to 2.39.

Figure10 illustrated two regions. First, safe region, i.e., the points lying inside of the failure assessment curve since the crack growth during applied load for linear elastic conditions is equal the resistance of material to fracture, whereas those outside of the curve are unsafe region. Second region unsafe region, the points lie outside of the curve, this shown fracture condition to propagate until complete fracture occurs.

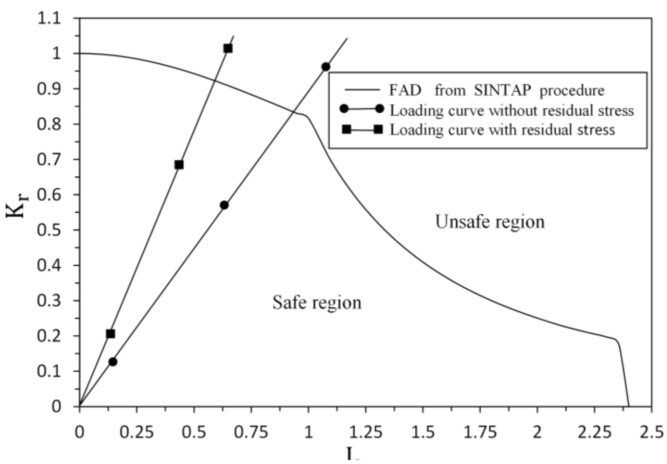

Fig.10. Failure assessment diagram for polyethylene pipe under pressure loading

The ratio of applied pressure 5MPa. to the limiting pressure 10 MPa. is Lr and is equal to 0.5, the stress intensity factor estimated in a pipe without residual stresses has a value of 2.505 MPa calculated using eq.11. The value of Kr is calculated from relation K_I / $K_{ic.}$ The Kr value is 0.64. The stress intensity factor with residual stress has a value 4.345 MPa obtained from using Eq.13 and corresponding Kr values is 0.94. The both points (0.5, 0.417) and (0.5, 0.725) are located in safe region, i.e. in the interior of the FAD curve. The applied loading to the PE pipeline is considered not dangerous and have an acceptable crack size.

In order to determine the values of L_r from FAD corresponding to crack initiation, we take intersect point between failure assessment curve and loading curve. Then, the plastic pressure corresponding to crack initiation calculated using relation (L_r * P_p). From fig.10, the values of L_r = 0.64 without residual stress and the calculated plastic pressure correspond to crack initiate is equal to 0.64 * 1301.51 = 832.97 kPa and L_r = 0.96 with residual stresses and the crack plastic pressure correspond to crack initiation 0.96* 1301.51 = 1249.45 kPa. Comparison these results, see that the plastic pressure corresponding to crack initiation will decrease due to effect of residual stress in PE pipe.

As is clear from the figure 10, the residual stresses are important factor and have a significant impact on identifying the types of state of area where the fracture or failure and the crack growth will occur. Figure 10, shows that the presence of residual stresses will transform the failure zone from elastic plastic fracture mechanics to brittle failure (linear elastic fracture mechanics).

Figure 11 shows the probability of failure for PE pipe under pressure loading only obtained by drawing the probability of failure calculated using Eq.17-27. As shown, the probability of failure calculated using uni-axial and multi-axial functions will increase by the affect of residual stresses. The uniaxial functions give lower values compared to multi-axial function and the probability of failure decreases with increasing radius. Table 3 shows comparison of probability failure at inside and outside radius of pipe. As note in table 3, the maximum values of probability of failure occur at inside part of PE pipe.

Table3. Probability of failure at inside and outside radius of PE pipe

Weibull Functions	Inside radius		Outside radius	
	Without residual stresses	With residual stresses	Without residual stresses	With residual stresses
Unia-axial	77.6	80.1	56.13	58.22
Multi-axial	82.21	85.62	63.1	67.32

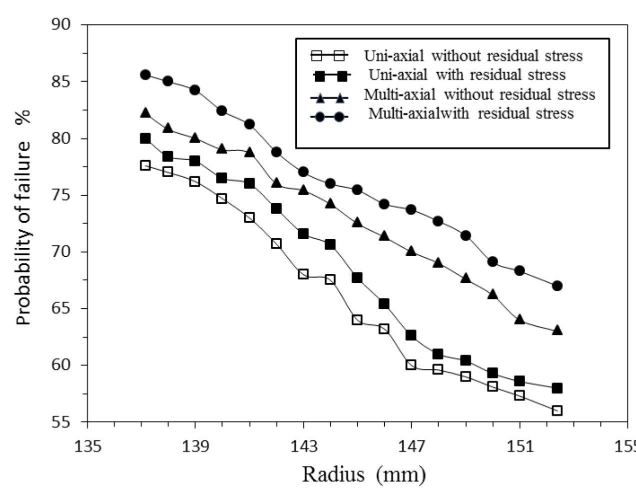

Fig. 11 Probability of failure of PE pipe under pressure loading

V. Conclusions

From previous discussion, the following conclusions are obtained:

1-The residual stress distribution in PE is non- linear and not uniform

2-The residual stresses increases the probability of failure of both uniaxial and multi-axial Weibull distribution functions

3-The residual stresses causes translation from elastic-plastic region on FAD to brittle region of linear elastic fracture mechanics.

4-A significant increase of the stress intensity factor for a pipe under action of residual stresses with in comparison with a case without residual stresses.

5-Weibul uni-axial distribution function give lower values compared to multi-axial functions.

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VII. References

[1] Fawad T., Nausheen N., Muhammad A. K., Rasheed A. B., "Failure analysis of high density polyethylene butt weld joint", Journal of Failure Analysis and Prevention, 2011.

[2] Hutar P., Sevcik M., Frank A., Nahlik L., Kucera J., Pinter G., " The effect of residual stress on polymer pipe lifetime", Engineering Fracture Mechanics, vol.108, pp. 98-108, 2013.

[3] Fangjuan Q., Lixing H., Yufeng Z. and Hongyang J., " Study on fracture properties of high-density polyethylene (HDPE) Pipe", Key Eng. Mat., vol.261, pp. 153-158, 2004.

[4] Dolas D. R., Jaybhaye M. D. and Deshmuk S. D., "Estimation the system reliability using Weibull distribution", International Proceedings of Economics Development and Research, vol.75, No.29, pp.144-148, 2014.

[5] Pluvinage G., Capelle J., Schmitt C. and Mouwakeh M., "Domain failure assessment diagram for defect assessment of gas pipe", Report of European Structural Integrity Society, International Conference on fracture (ECF 19), 2012, Italia.

[6] Sunwoong C. and Lawrence J. B., "Residual stresses in plastic pipes and fittings, I-methods for experiments analysis", Polymer (Korea), vol.21, No.1, pp.71-82, 1997.

[7] Rabia K. and Kamal C., "Reliability analysis of buried HDPE pipe", Journal of Engineering and Applied Science, vol.2, issue 4, pp.728-732, 2007.

[8] Joon S. L., Sang L. K. and Chang R. P., "Failure probability estimation of pressure tube using failure assessment diagram", Solid State Phenomena Trans., Vol. 120, pp. 37-42, 2007.

[9] Frank A., Pinter G. and Lang R.W., "Prediction of the remaining lifetime of polyethylene pipes after up to 30 years in use", Polymer Testing, vol.28, pp. 737-745, 2009.

[10] Alexander C. and Byoung H. C., "Crack initiation in high density polyethylene pipe resulting from chemical degradation", 11th Pan-American Congress of Applied Mechanics, Foz do Iguaçu, Brazil, January 04-08, 2010.

[11] Hutar P., Sevcík M., Zouhar M., Náhlík L. and Kucera J., "The effect of residual stresses on crack shape in polymer pipes", European Structural Integrity Society (ESIS) Conference, March 6, 2012, pp.486-496, UK.

[12] O'Connor C., Denton G. N., "The nature of polyethylene pipe materials", pipeline and gas Journal, vol.239, No. 12, 2012.

[13] Kong F. T., Lutfor R. K. and Hongshuang L.,"Reliability analysis of underground pipelines using subset simulation", International Journal of Civil, Structural, Construction and Architectural Engineering, vol.7, No.11, 2013.

[14] Jae J. H., Ho W. R., Yun J. K., Jong S. K., Young J. O. and Heung B. P., "Failure assessment diagram analysis of high density polyethylene pipes", Journal of Mechanical Science and Technology, vol.28, No.12, pp. 1-10, 2014.

[15] Mohammed Z., Shaju K. A., Mahadevan S., Vasudevan M., Vilas C. and Suri V. K., "Experimental and finite element analysis of residual stress and distortion in GTA welding of modified 9Cr-1Mo steel", Journal of Mechanical Science and Technology, vol.28, Issue 12, pp. 5095-5105, 2014.

[16] Withers P. J. and Bhafeshia H. K., "Residual stress nature and origins", Part2, Material science and technology, vol.17, No.1, pp. 366-375, 2001.

[17] Ramachndra M. D. S., "Assessing the significant of crack in structural components using fracture mechanics", IE (I) Journal–CV, vol.84, No.1, pp. 136-144, 2003.

[18] Ragupathy K., Ramesh K. and Hall D., "Effects of negative biaxial loadings and notch on failure assessment diagrams", J. Pressure Vessel Technology Vol.132, No.1, 2009.

[19] Feddersen C., "Evaluation and prediction of residual strength of center cracked tension panels", ASTM, STP 486, 1970.

[20] Hearn E. J., Mechanics of material, Vol.2, Pergamon Press, England, 1985.

[21] Yotaro Matsuo and Hideyuki Sato, "A probabilistic treatise of rupture of brittle rotating disc", Bulletin of the JSME, vol.22, No.172, 1989.

[22] Reedy D. V., Ataogl S., "Experimental analysis of buried high density polyethylene pipe", Turkish J. Eng. Env. Sci., pp.239-300, 2002.

[23] Bhattacharyaa P., "Study on Weibull distribution for estimating the parameters", Journal of the applied quantitative methods, Vol.5, No.2, 2010.

[24] Paritosh B., "A Study on Weibull distribution for estimating the parameters", Journal of Applied Quantitative Methods, vol.5, No.2, 2010.

[25] Designation: E 837 – 08, "Standard test method for determining residual stresses by the Hole-Drilling Strain-Gage method, ASTM International report, USA, 2008.
[26] Steve Lampman, Characterization and failure analysis of plastics, First Edition, ASM International, 2003.
[27] D 638 - 02a, "Standard test method for tensile properties of plastics, ASTM International report, 2003.

VIII. Nomenclature

a : Crack length mm.
b, c : Constants.
E: Modulus of elasticity MN/m^2.
r_i , r_o: Inner and outer radius of pipe mm.
r_h : The radius to the longitudinal center of the strain gage mm.
r_p : Plastic zone mean radius mm
r: Any radius of pipe mm.
D_i, D_o: Inner and outer diameter of pipe mm.
LEFM: linear elastic fracture mechanics.
EPFM: Elastic-plastic fracture mechanics.
t: Thickness of pipe mm.
K: Parameter constant.
$K_{mat.}$: Fracture toughness of metal MPa.$\sqrt{m}$.
K_T: Total stress intensity factor under mode I conditions MPa.$\sqrt{m}$.
$K_{I\,(int+res)}$: The stress intensity factor takes the effect of both internal pressure loading and residual stress.
K_r: Ratio of total stress intensity factor to fracture toughness of metal.
L: Length of pipe m.
L_r: Applied load normalized by limit load.
P: Pressure loading MPa.
P_{res}: pressure corresponds to measure residual stress at inner wall of pipe MPa.
m: Weibell modulus.
σ_r : Radial elastic stress MPa.
σ_t : Tangential elastic stresses MPa.
σ_{rr} : Radial residual stress MPa.
$\sigma_{\theta\theta}$: Tangential residual stress MPa.
σ_o : Weibull parameter MN/m^2.
σ_u: Ultimate strength of material MN/m^2.
σ_y: Yield strength of pipe material MN/m^2.
φ, ϕ: Angles between crack and coordinates.
β: Beta functions.
$\sigma_{max.}$: Maximum applied stress MN/m^2.
$R(\sigma)$: Probability of failure %.